FACHSCHAFT
MATHEMATIK

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Talk List

26.06.2015 - 27.06.2015
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Contents

1 Friday 10-12

1.1 TBA (Felix Boes)

1.2 Ein Geometrischer Beweis des Satzes von Hurewicz (Thorben Kastenholz)

Der Satz von Hurewicz ist ein wohlbekannter Satz aus der algebraischen Topologie, der tagtäglich Anwendung findet.

Satz. *Sei* (X, x_0) *ein* $(n-1)$*-zusammenhängender punktierter Raum, wobei* $n \geqslant 1$ *ist. Dann gilt für* $n = 1$ $(\pi_1(X, x_0))^{\mathrm{ab}} \cong H_1(X)$ *und falls* $n \geqslant 2$ *gilt* $\pi_k(X, x_0) \cong H_k(X)$ *für alle* $k \leqslant n$*. All dies Isomorphismen sind natürlich.*

Die Theorie der p-Stratifolds erlaubt es diesen Satz mit sehr geometrischen Mitteln zu beweisen. Das Entscheidende hier ist, dass p-Stratifolds eine Homologietheorie bilden, ähnlich wie Manigfaltigkeiten bis auf Bordismen eine Homologietheorie bilden. Diese ist für alle Räume natürlich isomorph zu singulärer Homologie, jedoch lassen sich p-Stratifolds in vielen Fällen wie Mannigfaltigkeiten händeln und erlauben so sehr geometrische Manipulationen die letzten Endes diesen Satz beweisen werden.

Mit ähnlichen Mitteln lässt sich noch das folgende Resultat beweisen:

Satz. *Sei* $n \geqslant 3$ *und* (X, x_0) *ein* $(n-1)$*-zusammenhängender punktierter Raum dann ist die folgende Sequenz exakt und natürlich:*

$$\pi_n(X, x_0)/2\pi_n(X, x_o) \xrightarrow{-\circ h_n} \pi_{n+1}(X, x_0) \xrightarrow{H_n} H_{n+1}(X) \to 0$$

Hier stehen H_n *für die entsprechende Hurewiczabbildung und* h_n *für die* $(n-2)$*-te Einhängung der Hopfabbildung* $h_2 \colon S^3 \to S^2$*. Für* $n = 2$ *gilt immerhin, dass* $H_2 \colon \pi_{n+1}(X, x_0) \to H_{n+1}(X)$ *surjektiv ist.*

1.3 Quillen's Plus-Construction and Algebraic K-Theory (Leon Hendrian)

2 Friday 12-14

2.1 Counting Covers of Elliptic Curves (Orlando Marigliano)

Riemann surfaces are one-dimensional complex manifolds, i.e. geometric objects that locally look like the complex plane. Their structure is much more rigid than e.g. the structure of a real surface, hence it is easier to classify the morphisms between them.

After refreshing the notions of Riemann surfaces, ramified covers, and elliptic curves, we turn to the problem of computing the number of ramified covers (i.e. morphisms) of Riemann surfaces into a fixed elliptic curve.

Using general covering theory, we can translate this geometric problem about surfaces into a combinatorial one about the symmetric group. The latter may be solved by algebraic methods, ranging from basic combinatorics in the symmetric group to the representation and character theory thereof.

We will see how to apply the technique of generating functions to attack general counting problems, and how it can help solve our problem.

If time permits, we will see that the generating functions we are interested in are *quasimodular forms*, a generalization of modular forms.

2.2 Spektraltheorem für Unbeschränkte Operatoren (Thomas Bodendorfer)

Ein Spektraltheorem gibt der Idee, dass man nicht nur Zahlen sonder auch (Differential)operatoren in Funktionen einsetzen kann und immer noch sinnvolle Ausdrücke erhält, eine mathematische Grundlage. In dieser Arbeit wird die Existenz eines solchen Spektraltheorems mit der Hilfe von C^*-Algebren bewiesen, und nicht wie sonst rein funktionalanalytisch.

2.3 Singular Integrals (Nikolay Barashkov)

3 Saturday 12-14

3.1 Loop Objects in Pointed Derivators (Aras Ergus)

TL;DR: *Derivators provide an abstract framework for homotopy theory. In particular, many statements from (classical) homotopy theory can be formulated and proven for certain kinds of derivators. My thesis is about one such statement, namely a "derivator version" of the fact that the loop spaces have a canonical group object structure in the homotopy category of pointed topological spaces.*

In some fields of mathematics, especially in homotopy theory and homological algebra, one wants to see certain kinds of morphisms (weak equivalences or homotopy equivalences of topological spaces in the first case, quasi-isomorphisms of chain complexes in the second case) as isomorphisms. Given a category $\mathbf{C}$ and a class W of "weak equivalences" which one wants to see as isomorphisms, one can, in certain cases which include the ones above, construct the "homotopy category" $W^{-1}\mathbf{C}$ equipped with a functor $Q\colon \mathbf{C} \to W^{-1}\mathbf{C}$ such that $Q(f)$ is an isomorphism whenever $f \in W$ and $(W^{-1}\mathbf{C}, Q)$ is universal with this property.

A question one could ask in this situation is whether $W^{-1}\mathbf{C}$ has all (co)limits if $\mathbf{C}$ does. The answer to this question turns out to be negative in most non-trivial cases.

In order to find a replacement for (co)limits in $W^{-1}\mathbf{C}$ one considers the following alternative description of (co)limits: Given a small category A and a category $\mathbf{C}$ which has all limits and colimits, there are adjunctions

$$\Delta\colon \mathbf{C} \leftrightarrows \mathbf{C}^A\colon \lim \quad \text{and} \quad \operatorname{colim}\colon \mathbf{C}^A \leftrightarrows \mathbf{C}\colon \Delta,$$

where $\mathbf{C}^A$ is the category of functors from A to $\mathbf{C}$ with natural transformations as morphisms and $\Delta\colon \mathbf{C} \to \mathbf{C}^A$ is the "constant diagram functor" which maps each object c of $\mathbf{C}$ to the functor which is contantly c on objects of A and constantly id_c on morphisms in A.

Now the idea is to replace $(W^{-1}\mathbf{C})^A$ by $W_A^{-1}(\mathbf{C}^A)$, where W_A is the class of morphisms in $\mathbf{C}^A$ (i. e. natural transformations) whose all components lie in W. This distinction between "diagrams in the homotopy category" and "diagrams up to pointwise weak equivalences" turns out to be very important and one in fact does have adjunctions

$$\Delta^{\mathrm{h}}\colon W^{-1}\mathbf{C} \leftrightarrows W_A^{-1}(\mathbf{C}^A)\colon \operatorname{holim} \quad \text{and} \quad \operatorname{hocolim}\colon W_A^{-1}(\mathbf{C}^A) \leftrightarrows W^{-1}\mathbf{C}\colon \Delta^{\mathrm{h}}$$

if $\mathbf{C}$ has all limits and colimits, where $\Delta^{\mathrm{h}}\colon W^{-1}\mathbf{C} \to W_A^{-1}(\mathbf{C}^A)$ is again a "constant diagram functor" similar to the one above.

From this point on many statements in homotopy theory and homological algebra can be shown purely formally using such adjunctions. The concept of a *derivator* provides an axiomatization of this phenomenon which describes the totality of “categories of coherent diagrams” and certain adjunctions between those.

For example, for a large class of derivators one can define a loop functor which corresponds to the loop space functor in the case of the homotopy theory of pointed topological spaces and to the shift functor in homological algebra. A well-known statement in homotopy theory is that loop spaces have a group object structure in the homotopy category (which is given by concatenation and inversion of loops). One can indeed show a “derivator version” of this statement, which is the topic of my thesis.